On the Value of Estimated State Uncertainty in Distribution System State Estimation

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Abstract

This paper investigates the importance of the estimation uncertainty in Distribution System State Estimation (DSSE). The study is carried out using data from a real implementation of DSSE in a medium-voltage distribution network in Switzerland, which is equipped with Distribution Phasor Measurement Units (D-PMUs). The study presents the computation of the uncertainty of state estimates and discusses the benefit of considering the estimated state uncertainty while analysing the DSSE results, which increases the reliability in correctly detecting key grid events, such as reverse power flows and line congestions.

1. Introduction

Distributed networks are experiencing an unprecedented changes due to increasing load and distributed generation causing bidirectional power flow [1]. The intermittent and unpredictable nature of Distributed energy resources (DERs) pose significant operational challenges to Distribution System Operators (DSOs), who are now seeking to increase the situational awareness of their networks to manage these uncertainties [2]. Distribution System State Estimation (DSSE) is expected to play a significant role in addressing these challenges by providing comprehensive insights into grid operations. Unlike Load Flow computations that must have all nodal powers, DSSE computes the complete grid state, i.e., voltages, currents, power flows at all nodes/lines, from a limited set of measurements. This capability allows DSOs to minimize investment costs by placing monitoring devices only at strategic grid nodes, such as Remote Terminal Units (RTUs) or Distribution-Phasor Measurement Units (D-PMUs). Moreover, DSSE offers the advantage of consolidating digital twin models of the distribution grid, including line parameters and topology, through cross-validation with real-time field measurements.

Due to the scarcity of measurements in distribution grids, often to ensure full grid observability the DSSE necessitates the use of pseudo-measurements, which are inherently less accurate than real-time measurements. While this approach enables DSSE implementation even with few real-time measurement deployed, it decreases the accuracy of some state estimates [3]. In such conditions, considering the uncertainties associated with the estimated grid states is important to avoid mistaken analysis. In existing literature, few studies have focused on DSSE uncertainty. The authors in [4] investigated the sources of uncertainty in voltage magnitude estimation, highlighting the influence of both voltage measurements and branch current accuracy on the results. In [5], the focus shifted to analytically examining the impact of flow measurements on branch current estimation, offering insights into meter placement strategies to achieve specific accuracy targets for monitoring. While these studies provide valuable theoretical frameworks, they lack a validation on real-world network data, and a demonstration on how these values can be used by DSOs in daily operations.

On this respect, the paper presents the results of a DSSE solution in a real medium-voltage (MV) network operated by Services industriels de Lausanne (SiL), a major DSO in Switzerland. SiL has installed the D-PMU-based grid monitoring system of Zaphiro Technologies in part of its distribution grid to enhance both power flow observability and fault location capabilities. A real-time D-PMU-based DSSE continuously estimates the MV network state. In this paper, the accuracy of the computed uncertainty of the estimated states is assessed using field data. Additionally, we discuss practical use cases where we show the importance of the estimated state uncertainty when analysing the DSSE results.

The paper presents first some theory on estimated state uncertainty computation using Linear Weighted Least Square (LWLS) algorithm. Then it describes the measurement infrastructure and the characteristics of the selected MV grid. Finally, it presents a detailed analysis on the uncertainty of the estimated state provided by DSSE.

2. Estimated State Uncertainty in Distribution System State Estimation

DSSE is a statistical procedure that allows to compute the complete state of a power grid (i.e., voltages and current flows in all nodes/branches) by combining measurements from a limited set of grid nodes with a model of the grid. Often in distribution grids the monitoring devices are not enough to guarantee full observability of the grid, so many DSSE solutions adopt pseudo-measurements, namely measurements with large uncertainties derived from historical data or forecasts [6].

In this paper, the focus is DSSE using D-PMUs and, when needed, pseudo-measurements in unmonitored nodes. In particular, we use the LWLS algorithm which is characterised by a linear measurement model linking the D-PMU measurements with the state variables represented by the voltages at each nodes [2]. It is well known that D-PMU-based DSSE has several advantages [7], the main ones being a non-iterative and numerically stable algorithm due to its linearity, and the coherency of the measurement set thanks to the synchronized measurements.

Beyond estimating the complete grid state variables, the DSSE also enables the computation of their uncertainties. These uncertainties are captured by the covariance matrix of the estimated states, whose diagonal elements represent the variances of the individual state variables. The covariance matrix is defined as:

$$cov(\hat{\boldsymbol{x}}) = \boldsymbol{G}^{-1} \qquad (1)$$

where $\hat{\boldsymbol{x}}$ is the estimated state and $\boldsymbol{G}$ is the well-known Gain matrix, as explained in [2]. Concerning the other estimated electrical quantities (i.e., nodal/branch currents/powers), their uncertainties can be derived from the covariance matrix of the state variables using the error propagation principles. If we consider an estimated quantity q, we define the uncertainty u_q of its estimated value $\hat{x}_q$ as:

$$u_q = 3\sigma_{\hat{q}} \qquad (2)$$

where $\sigma_{\hat{q}}$ is the standard deviation of the estimated quantity. Therefore, the uncertainty u_q corresponds to a 99.7% confidence interval and means that there is a 99.7% probability that the true value falls within the interval $\hat{x}_q \pm u_q$.

3. Grid structure and D-PMU placement

The study presented in this paper is based on a real MV feeder in Switzerland operated by Services Industriels de Lausanne (SiL). The feeder supplies a small residential area in Lausanne and has been equipped with 6 D-PMUs of Zaphiro Technologies. Figure 1 illustrates the single-line diagram of the selected MV feeder and the placement of the D-PMUs. The feeder is radial, operated at a rated voltage of 11.5 kV, and consists of 16 MV lines, 12 MV/LV secondary substations, and 4 switching substations. The D-PMUs measure branch current phasors at every departure using Rogowski coils. The only voltage measurement is provided by the D-PMU at the primary substation which is connected to the voltage transformers on the substation busbar.

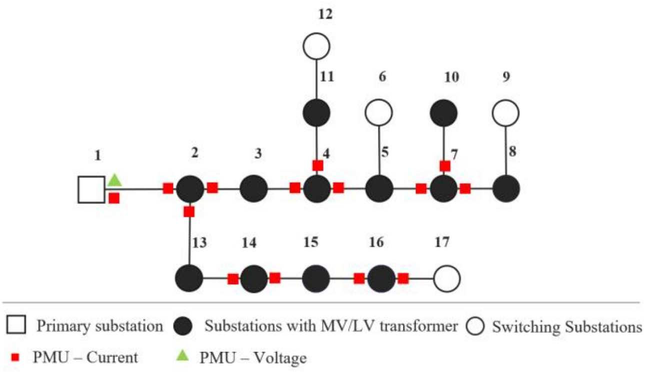

Figure 1: Single line diagram of the selected MV feeder.

To illustrate the grid operating conditions considered in this paper, Figure 2 shows the time profiles for a full day at the primary substation for voltage and active power flows, with positive flows exiting the bus. As we can observe, the grid is mostly passive, apart from few instants around noon where there is a reverse power flow due to the production of PV panels.

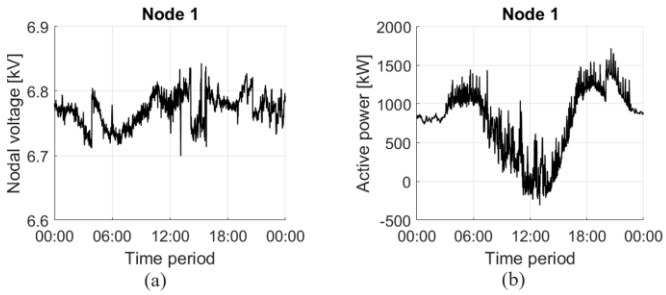

Figure 2: Time profiles for one day of voltage (a) and active power flows (b) at the primary substation.

4. Methodology to assess the accuracy of the estimated state uncertainty

The objective of this study is twofold: (i) to assess the accuracy of the computation of the estimated state uncertainty, and (ii) to show the usefulness of considering the estimated state uncertainty while analysing the estimated grid state.

The assessment process involves two key steps. The first step consists in obtaining a reference grid state to serve as a baseline for the rest of the analysis. Since this is a real grid installation, the true state of the grid is inherently unknown. However, by utilizing all D-PMU measurements the grid becomes fully observable [2], thus ensuring highly accurate estimates of all the state variables. Therefore, the state estimated in this way can be used as the *reference state* throughout the paper for performance assessment.

The second step consists in running the DSSE using only four D-PMUs in nodes 1, 4, 7, and 14, as shown in Figure 3. This configuration simulates a realistic scenario for an extensive deployment, where only 25% of the grid nodes are measured. To maintain grid observability under these conditions, nodal current pseudo-measurements are incorporated at unmonitored MV/LV secondary substations. The DSSE results, including both the estimated variables and their associated uncertainty, are then compared to the reference state obtained in the first step.

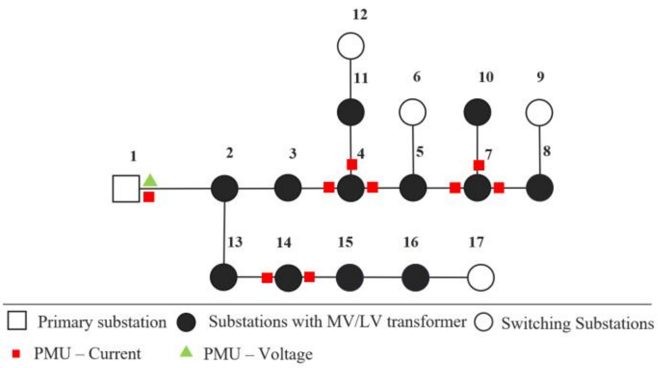

Figure 3: Single-line diagram illustrating the considered measurement configuration.

For the purpose of this study, we used a time window of one day at a resolution of one measurement per minute (despite D-PMUs report synchrophasors at 50 frames-per-second), resulting in a total of 1,440 time-steps. The assessment focuses exclusively on positive sequence voltages and currents, whereas powers are the sum of the three-phases.

Furthermore, the measurement variances of real-time measurements are computed according to the accuracy class of the voltage and current sensor as shown in [7]. Concerning the pseudo-measurements, they are significantly less accurate than the D-PMU measurements, since their values and variances are derived from very limited information including the rated power of the transformers. Pseudo-measurements are used at unmonitored nodes in Figure 3, namely nodes 2, 3, 5, 8, 10, 11, 13, 15, 16. The switching substations are zero-injection buses that are integrated in the DSSE as equality constraints, as presented in [7].

To evaluate the performance of DSSE in estimating a given state variable, three metrics are chosen:

- the *reliability* (Rel) of the DSSE uncertainty interval, defined as the percentage of timestamps where the reference state falls within the uncertainty intervals;
- the *uncertainty* (u) of an estimated quantity, defined in equation (2) as 3 times the standard deviation of the estimated quantity;
- the *absolute error* (AE) of an estimated state variable x_k at time t, defined as:

$$AE = \left| x_{k,t}^{est} - x_{k,t}^{ref} \right| \qquad (3)$$

For AE and u, we compute the median value and the 95th percentile (P95) on all time-steps in order to capture the central tendency and extreme cases, respectively.

5. Accuracy assessment of the estimated state uncertainty with real field data

In this section, we evaluate the performance of the DSSE by analysing its ability to estimate both nodal and branch quantities and the associated uncertainties.

5.1 Uncertainty of estimated line power flows

In the DSSE framework, buses/branches can be classified as either "monitored", namely directly measured with a real-time measurement, or "unmonitored". For the measurement configuration shown in Figure 3, the unmonitored lines are the ones connecting nodes 2-3, 2-13 and 15-16, while the other lines are either monitored or connected to zero-injection buses. To evaluate DSSE performance in estimating line power flows, Figure 4 and Figure 5 show the active and reactive power flows for the monitored line 3-4 and the unmonitored line 2-13, respectively.

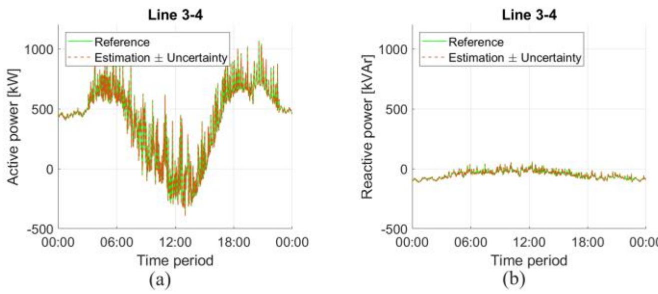

Figure 4: Time-series of the estimated active (a) and reactive (b) powers on the monitored line 3-4.

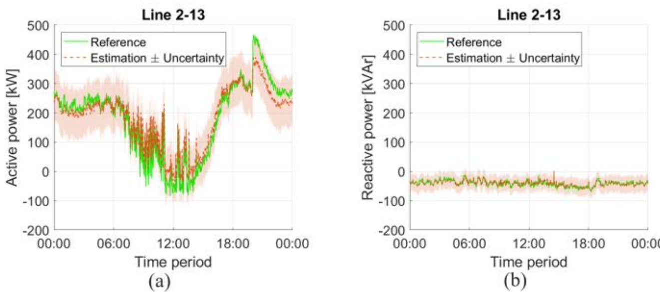

Figure 5: Time-series of the estimates active (a) and reactive (b) powers on the unmonitored line 2-13.

Table 1: Performance metrics of DSSE estimations for active powers in the three unmonitored lines.

Line	Rel [%]	AE [kW]		*u* [kW]	
		median	P95	median	P95
2-3	99.9	18.3	60.8	85.8	86.3
2-13	99.9	27.3	57.7	85.6	86.1
15-16	99.4	18.2	36.5	52.2	89.4

Table 2: Performance metrics of DSSE estimations for reactive powers in unmonitored lines.

Line	Rel [%]	AE [kVAr]		*u* [kVAr]	
		median	P95	median	P95
2-3	100	4.3	13.2	29.4	30.3
2-13	100	2.6	8.2	28.7	29.2
15-16	99.9	1.7	6.8	18.5	29.5

Figure 4 shows that both active and reactive power flows in the monitored line 3-4 are estimated with high accuracy, since the estimated value perfectly follows the reference. The uncertainty band is not visible since it is very narrow of about ±10 kW. The same remarks apply for all the monitored lines. In the unmonitored line 2-13 in Figure 5, the estimated power flows still follow quite closely the reference value, but there are periods, such as from 20:00 to midnight, where the estimated active power slightly diverges from the reference. This is expected since it is an unmonitored line where the estimated power flow is affected by the inaccurate pseudo-measurements in nodes 2, 3, and 13, which cannot perfectly capture real-time grid dynamics due to their dependence on non-real-time information. However, we can observe that the reference value remains within the boundaries of the uncertainty interval, showing that considering the uncertainty is highly valuable when using the DSSE results.

Table 1 and Table 2 present the metrics defined in Section 4 for the three unmonitored lines concerning respectively active and reactive power flows. The *reliability* (Rel) of the estimates is close to 100% for all the lines, demonstrating that the interval defined as "estimated value ± uncertainty" is able to contain the reference value, which is the target. Both active and reactive power flows are accurately estimated despite the lines being unmonitored, with median value of the *absolute error* (AE) that is respectively < 30 kW and < 5 kVAr. It is also interesting to observe that for lines 2-3 and 2-13 the uncertainty interval is quite constant along the day, which is visible since the median and the 95th percentile (P95) of the uncertainty have similar value. On the contrary, for line 15-16 the median and the P95 are different because the uncertainty varies along the day due to changes during the day in the uncertainties of pseudo-measurements in the unmonitored area composed of nodes 15 and 16.

It is also worth observing that in line 2-13 in Figure 5, the actual active power flow reverses direction around 10:00 while the estimated value remains positive. If we relied solely on the estimated value, this inversion would go unnoticed. However, the lower bound of the uncertainty interval includes also negative values, highlighting a notable probability of power flow reversal and demonstrating the value of incorporating uncertainty when analysing the DSSE data.

5.2 Uncertainty of estimated voltage and nodal power flows

The results of voltage and nodal apparent power flow are shown for the monitored node 1 in Figure 6 and two unmonitored nodes: node 2 in Figure 7 and node 5 in Figure 8.

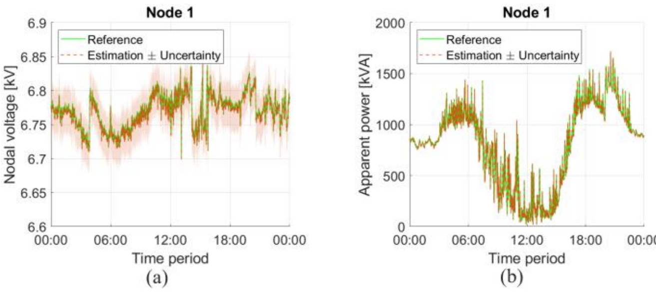

Figure 6: Time-series of the estimated voltage (a) and nodal apparent power flow (b) at the monitored node 1.

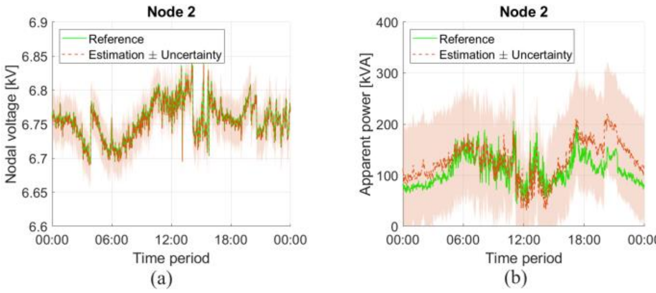

Figure 7: Time-series of the estimated voltage (a) and nodal apparent power flow (b) at the unmonitored node 2.

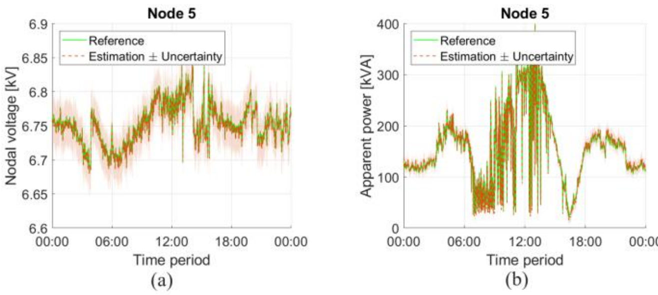

Figure 8: Time-series of the estimated voltage (a) and nodal apparent power flow (b) at the unmonitored node 5.

Both monitored and unmonitored nodes exhibit comparable voltage uncertainties, which are approximately 30 Volt or 0.5% of the nominal voltage. This indicates that the measurement error (related to 0.5 class voltage transformers) is the primary contributor to voltage estimation uncertainty and that other sources of error, such as power flow estimation errors, are negligible in comparison.

Regarding the nodal power estimates, the monitored node 1 has an uncertainty of approximately ±20 kVA. Similar uncertainty is observed for all the other monitored nodes 4, 7, 14. The unmonitored node 2 exhibits a significantly higher uncertainty of about ±100 kVA, whereas node 5 has an uncertainty of approximately ±15 kVA, which is in the same range as the monitored node. The difference in uncertainties between nodes 2 and 5 can be explained by analysing the measurement configuration in Figure 3. Node 2 is part of a grid region delimited by D-PMUs with multiple unmonitored nodes, namely the region with nodes 2, 3, and 13. In this configuration, the DSSE relies also on high-variance pseudo-measurements to estimate the power flows of the unmonitored nodes in the area, leading to larger uncertainties. Such regions are referred to as regions with low local measurement redundancy. On the contrary, node 5 is situated in a region with high local redundancy, since it lies between nodes 4, 6, and 7, which are either equipped with a D-PMU or a zero-injection node. The availability of multiple accurate real-time measurements around node 5 allows the DSSE to accurately estimate the nodal power flow at this node.

Table 3 presents the metrics defined in Section 4 for all unmonitored nodes, where the nodes are grouped per grid area. For areas with only one unmonitored node (nodes 5, 8, 10, and 11), the nodal power estimates are very accurate: AE of few kVA and uncertainty < 20 kVA since these nodes are in high local redundancy areas. In contrast, the low local redundancy areas, namely the area with nodes 2, 3, 13 as well as the area with nodes 15 and 16, exhibit higher AE. However, we can observe that the reliability of the estimates remains high, because the DSSE correctly increases the computed uncertainty in areas with low local redundancy and thus the reference value remains within the uncertainty boundaries in >99% of the cases.

Table 3: Performance metrics of DSSE estimations for nodal apparent power flows in unmonitored nodes.

Node	Rel [%]	AE [kVA]		u [kVA]	
		median	P95	median	P95
5	100	2	6	13.5	17.2
8	100	0.03	1.7	1.8	11.1
10	100	3.8	6.6	13.2	17.8
11	100	0.2	0.3	4.4	6
2	100	22.6	55.6	102.5	103.6
3	99.9	18.6	61.3	85	85.7
13	99.9	24.1	56.6	85	85.7
15	99.9	17.2	36.2	52	89.3
16	99.4	18.2	36.5	52	89.3

6. Conclusion

The scarcity of measurement devices in distribution grids requires DSSE to use inaccurate pseudo-measurements that affect the accuracy of some state estimates. By leveraging real data from a D-PMU-based DSSE implemented in a real distribution grid in Switzerland, this study has demonstrated the benefits for DSOs of considering also the estimated state uncertainty when performing studies and taking decisions based on DSSE data. We demonstrated that, while sometimes the estimated value slightly diverges from the true value, the estimated value remains most of the time within the boundaries of the uncertainty interval, showing that the uncertainty is correctly computed. The use of the uncertainty of estimated states increases reliability in correctly detecting key grid events, such as reverse power flows or line congestions, that can be overlooked when relying solely on estimated values.

7. References

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